Numeracy : An Annotated Bibliography for Teachers and Schools

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Preface

The Numeracy Task Force based its programme of work, its two reports and its policy recommendations on careful study of the world's knowledge bases about mathematics education and mathematics teaching. Some of this material is mentioned in our two reports *Numeracy Matters* (published in January, 1998) and *The Implementation of the National Numeracy Strategy* (published in July 1998), but inevitably a large amount of material that might be useful to schools has hitherto remained unpublished. This annotated bibliography brings this material to the attention of schools and teachers, for your possible use. We have attempted to focus primarily on material about primary/elementary schools, and have of course focussed, although not exclusively, on British material. Whilst the great majority of the material is based upon research, some well known more polemical policy and ‘think’ pieces are also included. Articles in this bibliography from academic journals can be obtained through University libraries or by inter-library loan from University and public libraries. Books mentioned can be ordered through University and from public libraries if they are not on the shelves. Reports from official bodies or from public bodies such as OFSTED can be obtained by contacting the publications department of the organisations concerned if there is any difficulty getting them.

We hope that the material will be of use in helping to reach our ambitious mathematics targets for British children.

Professor David Reynolds
Chair, Numeracy Task Force
This annotated bibliography is not designed to be exhaustive. Rather, we have attempted to provide the reader with a list of resources to draw upon to provide an introduction to the topics listed below. There may be duplication of some entries under different topics, as some articles/books address more than one of the issues. Where this happens, after the first presentation of an entry, subsequently only the title of the entry is given.

The following topics are covered:

1. The Importance of Numeracy

2. Effective Teaching of Mathematics
   a. Effective Whole-class Teaching
   b. Small Group Work and Pupil Grouping
   c. Teacher Beliefs and Subject Knowledge
   d. Novice-Expert Studies, Constructivism and Other Miscellaneous Effective Teaching Research

3. Mathematical Content and Curriculum

4. Child Development and Mathematics

5. ICT and Calculators in Mathematics Teaching

6. International Studies of Mathematics Achievement

7. Teacher Training and Professional Development in Mathematics

8. The Effects of Parental Background, Gender and Ethnicity upon Mathematics Achievement
9. Pupil Attitudes and Self-Concept in Mathematics
1. **The Importance of Numeracy**


A comparative study of the numeracy skills of adults in seven countries (UK, France, Netherlands, Sweden, Japan, Australia and Denmark) carried out by market research companies with a representative sample of over 650 respondents in each country, who were given 12 numeracy tasks. Respondents from the UK performed least well of all participating countries, scoring significantly lower than the second worst performing group, Australians. Only one in twelve UK respondents managed to answer all 12 questions correctly. Japan emerged top, followed by France and Holland. Questions UK respondents found most difficult mirrored problem areas in other countries, i.e. using fractions and percentages and calculating areas.


This report examines the main findings of research undertaken to assess the level of demand for reading, writing, oral communication and numeracy skills at work. Results are drawn from surveys conducted for the Adult Basic Literacy and Skills Unit in 73 Training and Enterprise Councils in England and Wales. The report describes the different skills needed for different jobs. In larger and growing companies the demand for basic skills has increased over time. Results indicate that possession of sound basic skills is crucial for attaining employment.


This report, based on data from the longitudinal National Child Development Study, provides clear evidence that numeracy skills are important. People without numeracy skills suffered worse disadvantage in employment than people with poor
literacy skills alone. They left school early, frequently without qualifications, and had more difficulty in getting and maintaining full-time employment. The jobs entered were generally low grade with limited training opportunities and low pay and prospects. It was also found that numeracy problems were often not detected early in school, and that teachers had more problems detecting numeracy problems than they had detecting literacy problems.


Using a representative sample of 1650 members of the 1970 British Cohort Study at age 21 (1992), the basic skills of young adults were tested using data collected from an interview and a literacy and numeracy test. It was found that a larger group had problems with numeracy than did with literacy. Poor numeracy was related to poor literacy, but poor numeracy added another independent set of difficulties to those already present through poor literacy. Poor numeracy was associated with unskilled family backgrounds and parents who had failed to gain any qualification. School leavers with poor numeracy and literacy skills were more likely to have experienced unemployment and problems at school.


Using data on respondents from the National Child Development Study interviewed at age 23 (1981), 13% reported difficulties with literacy or numeracy. Of these, 39% reported problems with numeracy. More people reporting basic skills problems were unemployed than in the cohort as a whole (15% v. 9%), and most of those in work were working in less skilled jobs. They were less likely to own their own house and more likely to live with relatives or in council housing. They had lower wages than people not reporting basic skills problems and were more likely to be receiving
some form of benefit. A large proportion of respondents with basic skills problems reported not having received any help at school.
2. **Effective Teaching of Mathematics**

2a. **Effective Whole-Class Teaching**


In this article the author reviews effective teaching research and points out its relevance to mathematics teaching, with a view towards suggesting future areas for research. The author emphasises the problems involved with using theories of individual psychological development in classroom settings and emphasises the utility of whole-class teaching in this respect. He points to a number of factors found to be effective, such as an academic emphasis and clear and well-structured presentations. Other elements discussed include classroom organisation and grouping, seatwork (individual practice) and teaching cognitive skills.


This book chapter reviews process-product research linking teacher behaviour to student outcomes, in particular academic achievement. The review gives a summary of all the major studies undertaken up to the publication of the article, but also attempts an integration of the findings from these studies. Results show a consistent negative relationship between student achievement and pupils working on their own. Active teaching (whole-class interactive) is positively related to outcomes. Important factors are divided into giving information, questioning students and providing feedback. Significant factors in giving information are a clear structure using advance organisers, outlining the content of the lesson and summarising subparts and the whole of the lesson, a degree of sequential redundancy in presenting information, clarity of presentation, teacher enthusiasm and rapid pacing if the subject is basic skills or somewhat slower pacing if more complex material is presented. As for questioning the students, it appears to be necessary to vary the difficulty of questions depending on
content, to use both higher and lower cognitive level questions, to use clear questions, and to vary wait time after questions in accordance with the difficulty of the question. Feedback to correct responses should be overt but usually factual, feedback to partially correct responses should be affirming of the correct part and prompt the correct response (e.g. give clues) for the incorrect part, and feedback to incorrect responses should be overt but avoid personal criticism. Teachers should try to elicit a correct response from the student but avoid pointless pumping when the student is too confused to provide the correct answer. The authors also provide some limitations of this type of research.


A review of research on classroom management. The chapter is divided into six major units. In the first section the author discusses the problem of order in classrooms, in order to provide a conceptual framework for understanding the nature of the management function in teaching. The second section is directed to the issue of how life in classrooms is organised. The third section examines the programs of action that are embedded in classroom contexts, including both social participation structures and academic work structures. The fourth section is focussed on the issue of how order is achieved in the classroom with particular emphasis on how activities are established at the beginning of the year, how lessons are accomplished and how classroom management affects, and is affected by, academic work. The fifth section concentrates on the nature of misbehaviour in the classroom, and the function of teachers' interventions to stop misbehaviour. The article concludes with a summary of the major themes emerging from the review.

This article describes a decade of classroom research carried out at the University of Leicester. The ORACLE project began in 1975 and consisted of two main projects. The first study was a longitudinal process-product study of teaching and learning in the junior age. The second study concentrated on collaborative group work. The major findings of both studies are described. In the first study it was found that pupils worked on their own for almost 60% of the time. Teachers only addressed the whole class for 15% of the time, but this small amount of time accounted for nearly three-quarters of the contact the teacher had with the pupils. When working with individual pupils, 80% of the teacher's exchanges were to do with task supervision and routines. In the second study teachers were given inservice training on collaborative group work. These teachers using collaborative group work were then compared to control group teachers who used class discussion followed by individual work. It was found that in the experimental group the proportion of lessons involving extended discussion was 35.9%, compared to 19.2% for the control group.


A classic whole-class interactive mathematics teaching experiment successfully implemented in fourth-grade classrooms in the US. Observational data showed that the 40 teachers involved implemented the programme, and comparison of pre- and post-test results showed that pupils taught by treatment teachers generally outperformed pupils in control groups.


Description of the research which led to the development of the active teaching mode. The book first sets out the naturalistic study which was conducted to determine whether it was possible to identify teachers who were consistently effective or ineffective across groups of pupils as measured by outcomes on the Iowa Test of Basic
Skills. Variables found to be related to effectiveness were: large-group instruction, generally clear instruction and availability of information to students as needed (process feedback in particular), a non-educative and relaxed learning environment which was task-focused, high achievement expectations and classrooms which were relatively free of major behavioural disorder. Teachers who obtained high student achievement test scores were active teachers who gave a clear, meaningful presentation of what was to be learned, provided developmental feedback when it was needed, structured a common seatwork assignment and responded to individual students' need for help. On the basis of these findings a training program for teachers was developed in order to test whether using these active teaching methods would benefit students if teachers changed their behaviour in accordance with them. This research was carried out in primary schools, and clearly supported the validity of the method; pre- and post-test comparisons showing that pupils in the experimental group outperformed control group students and also showed more positive attitudes towards mathematics. Similar, though less strong, results were obtained when in a second study a measure of verbal problem solving rather than a standardised achievement test was used.


Using data from the Second International Mathematics Study (SIMS) the influence of mathematics teaching practices as described in SIMS teaching scales (based not on classroom observation but on teacher questionnaire data) on class-level mathematics achievement was examined. Overall, 'show and tell' (stating properties or definitions; use of abstract knowledge and facts; emphasis on use of examples and empirical demonstration) and 'inference' (use of two and three dimensional drawings and objects, figures and examples and an informal approach based on inductive reasoning) predicted higher achievement, while 'comparison' (teaching abstract concepts by comparing them to other abstract concepts), 'eclectic' (an open, informal and eclectic approach in which trial and error as well as knowing why rules work are
features) and 'abstract' (a focus on memorisation of rules and formulae) practices predicted lower achievement.


This edited book provides a useful practical perspective on teaching numeracy in the primary years, based on a model which stresses the importance of actually teaching children (rather than the facilitating role popular following the Cockroft Report). The model consists of three components: 1. teaching, the role of the teacher being one of instructing, modelling, explaining, questioning and narrating; 2. making sense, which always needs to involve talk; 3. practice, necessary to aid memorisation. Within this framework contributors discuss developing numeracy in the early years, a teaching approach described as reading maths which sees maths as a language rather than as a body of knowledge, the role of calculators, number operations and procedures, bilingual children and numeracy, play and number and assessing numeracy.


A classic study of school effectiveness across a range of outcomes including mathematics achievement, which also includes valuable material on effective teaching of mathematics. Factors associated with mathematics ‘gain’ over time include teachers questioning behaviour, class organisation, use of textbooks, curriculum structure and assessment policy.

Review of the experimental whole-class teaching approach introduced in Barking & Dagenham, and the reasoning behind the strategy. Emphasis is on keeping the whole class more closely together in attainment. The greater part of the lesson is devoted to a teacher-led question and answer session. Mental strategies are emphasised over written algorithms in the lower year of primary school. Individual work and small group work are also used, but sparingly.

2b. Small Group Work and Pupil Grouping


The purpose of this study was to examine high-achieving students' interaction and performances on complex mathematical tasks as a function of homogeneous versus heterogeneous pairings. Participants were third and fourth graders, who had been trained in and had routinely practised constructive peer-tutoring interactions and who had experience of working individually on performance assessment. 10 high achievers working with a high achieving and with a low achieving classmate on performance assessment were videotaped. Results indicated that homogeneous dyads operated more collaboratively, generated greater cognitive conflict and resolution and produced better quality work.
Teaching and Teacher Education 3(4): 299-313.

British Journal of Educational Psychology 65: 211-225.

This study of 440 year 6 children looked at the effects of mixed gender and ability compositions on students' behaviours and interactions in structured work groups, using a behaviour observation schedule and a learning outcomes questionnaire. Children worked on class-based activities in small groups, which were structured so that all members had to cooperate in order for the group to achieve its academic objectives. The effects of different ability and gender compositions on group members' behaviour and interactions proved minimal. It appeared that as group members had more time to work together they became more responsive to each other's needs and gave more explanations to assist each other's learning so that all groups achieved comparable learning outcomes.


A review of the literature on small group cooperative learning, as well as discussion of the potential advantages and disadvantages of this method, and the way small-group learning can fit into a teaching-for-understanding framework. A number of different models are proposed in which small group work is integrated into whole-class interactive teaching models.

Review of research on small-group learning. While there is clear evidence that small-group instruction in non-set groups can facilitate student achievement, especially in basic skills, as well as supporting more favourable attitudes towards peers and subject matter, the authors argue that there is a lack of data on process factors and on the prerequisites for effective small-group instruction (the 'it depends' factors). Setting has been found to be less effective, however, in that the lower ability students grouped in lower sets tend to perform less well than when placed in heterogeneous classrooms. The same seems to hold for within-class ability grouping. Cooperative small-group learning has, however, been found to be highly effective in a number of studies, peer interaction helping students to construct mathematical knowledge. However, a number of problems do exist, in that once again it is the highest achievers (who tend to dominate group interaction) who seem to benefit most from this form of small-group learning. It would also seem that boys (who are more active in these groups) benefit more than girls. The authors present a number of recommendations for future research.


This article reports a naturalistic study of small-group instruction in elementary schools. Three school districts from three cities in three Midwestern states were studies. Observations (N=206) of entire mathematics periods were made in the classrooms of 33 teachers (primarily in grades 4, 5 and 6) in 21 schools. Six types of small-group formats were found: whole class ad hoc (whole class teaching with ad hoc formation of small groups), two groups, three groups, heterogeneous work groups (collaborative learning) and individualised grouping. It was found that compared to teachers using grouping as the main format of the lesson, teachers who mainly taught the whole class spent substantially more time on educationally important teaching functions such as lesson development and review, while teachers who mainly taught small groups or individuals spent more time on transitions and non-mathematical
activities. In whole class lessons more time was spent on higher order mathematics and use of manipulatives, but students working on individual work in whole class settings were more often off-task than those working in small groups.


Using a longitudinal data set of 1477 students in 48 classes (primarily grades 4 - 6) in Northern California, it was found that within-class assignment to ability groupings was influenced by pupil gender over and above their performance in maths, with boys tending to be assigned to higher ability groups. Girls with high mathematical aptitude were less likely to be assigned to a high set than boys with similar aptitudes. Within-class grouping was found to have no effect on pupil performance compared to whole class teaching.


The effects of two types of active teaching and learning on the mathematics achievement of 1736 fourth, fifth and sixth grade students in 81 classrooms was compared. The first was a whole-class setting in which ad-hoc remediation and grouping was used, the second was a two-grouped model that accommodated student diversity through fixed ability groups. Nine matched schools were randomly assigned to the two treatment conditions and teachers were trained in the two methods. It was found that whole-class ad-hoc teachers provided students with more development time, more individual attention, and twice as much assessment of understanding. Less time was spent on transition in the whole-class ad-hoc groups.

A study of small group cooperative learning among 112 fourth-graders from 4 classrooms. Generally pupils reported changed (more positive) behaviour after increased experience in small groups. No gender differences were found, small group work seemingly not contributing to gender stereotyped behaviour.


An experimental approach designed to enhance pupils' cognitive processing was tested. The acronym stands for the lesson phases in the approach. Lessons start with the teacher introducing new concepts to the whole class. Students then take turns asking and answering questions in small groups. The questions are of three types: Comprehension questions, Strategic questions and Connection questions. The process emphasises teaching pupils metacognitive processes such as using diagrams and tables to solve word problems. In a test in 6 seventh grade classrooms (N=164) which were compared to 3 control classrooms (N= 101) the approach was found to significantly enhance the performance of high ability and middle ability pupils, but not that of low ability students.


A study of 306 3rd through 6th grade highly able pupils, who started off with far more mathematical knowledge than their peers, found that these pupils made very strong progress when placed in a flexibly-paced program (individualised learning pace) as opposed to an 'age appropriate' curriculum.

A study of cooperative small group work in 3 classrooms in grades 3 to 5. After training teachers and students, lessons were videotaped to study pupil behaviour. It was found comparing pre- and post test scores that giving explanations, receiving explanations and giving and receiving other help were positively correlated with achievement while receiving no help after questioning was negatively related to achievement.


In this chapter the authors review a number of factors contributing to mathematics teaching and learning. In a first section they discuss theories of mathematics learning, with particular emphasis on cognitive theory. Subjects covered include analysis of pupil errors and the organisation of memory and metacognition. Implications for teaching are given. In a second section mathematics teaching is discussed. Topics include assessment, allocated and engaged time, the developmental part of the whole-class interactive lesson, instruction in small groups, teachers' planning and decision making, teaching strategies and coping with individual pupil differences.


Two action research studies of a fourth-grade classroom in which group grading was used in an effort to increase cooperation among group members and the mathematics test scores of low achievers in work groups of 3-5 members. It was found that group grading was an effective strategy for students who enjoyed group work, but not for students who preferred individual work (and individual grading).

This study examined the relationship between academic task values (for mathematics and language) and perceptions of personal social satisfaction for children in classrooms using a cooperative, interactive learning structure compared to children in regular classrooms. 162 children in six form 2 class (aged 13-15) participated. Three classes were taught using cooperative learning strategies. Using questionnaire scale results, it was found that task values for engagement in mathematics and language were higher, and perceived costs lower in classrooms using a cooperative goal structure. This was associated with higher social satisfaction in students.


Review article on task-related pupil behaviour in small cooperative groups. Factors consistently related to individual pupils’ achievement included giving content-related explanations (positive relation), receiving a lower level of help than is requested (negative relation) and off-task discussion (negative relation). Homogeneous, medium-ability groups and groups with a moderate range of ability encouraged active participation by all members, which was not the case for some other configurations (e.g. heterogeneous groups). The author presents a number of recommendations for influencing interaction in small groups.

2c. Teacher Beliefs and Subject Knowledge

This study of effective teachers was conducted in 6 schools known to be effective in numeracy, in 3 LEA's. Three teachers most likely to prove effective were selected for case studies in each school, as well as 3 teachers in 5 other schools. Teachers taught in years 1-6. Numeracy tests were used to establish pupil gains over two terms. Teachers could then be classified by effectiveness (highly effective, effective, moderately effective). Looking at teacher beliefs it was found that highly effective teachers were characterised by connectionist beliefs, which involve beliefs in what it means to be numerate (e.g. confidence and ability in mental methods, selection of methods based both on the operation and the numbers involved and awareness of links between different aspects of the mathematics curriculum, and beliefs about pupils and how they become numerate (e.g. through interpersonal activity, through being challenged and struggling to overcome difficulties and the belief that most pupils have strategies for calculating but the teacher has to help them refine their methods, pupil misunderstandings need to be recognised, made explicit and worked on). This orientation is contrasted to 'transmission' and 'discovery' orientations used by some of the less effective teachers. These orientations also have consequences for teaching practice, as they influence teachers’ beliefs on how best to teach pupils to become numerate. Teacher's pedagogic content knowledge was also studied, the nature of that knowledge proving more important than formal qualifications. Extended professional development programs were perceived by teachers as important to the development of their beliefs. This was less the case for initial teacher training.


This review of research presents short, practical discussions of a wide range of topics, such as children's early competency with number, the relationship between 'figuring it out' and 'knowing by heart', problems involved in moving from practical work to formal work, addressing common misconceptions, careful choice of examples, effective questioning, the quality and quantity of praise, the importance of teachers'
knowledge of pupils’ attainment, the need to introduce pupils to a wide range of problem-solving situations, how to increase success in problem-solving, the effect of pupil attitudes and beliefs, use of calculators, gender, use of ICT, use of cooperative small group work and the use of setting.


This book reports on a (qualitative) study designed to investigate reception teachers' pedagogical knowledge. It was found that at the heart of teachers' pedagogical subject knowledge lie subject content knowledge and knowledge of their pupils conceptions. It was found that teachers' classroom practice (as determined through classroom observation) was highly diverse, and that teachers seemed unaware of the rich informal knowledge children brought to school with them (as determined by criterion-referenced instruments designed to test this knowledge administered to pupils), a fact that raises some questions about the adequacy of teachers' subject knowledge. It was also found that teachers who had complex views about children's learning didn't necessarily translate these into practice, suggesting that without clear subject content knowledge neither sophisticated theories concerning children's learning nor scaffolding approaches will necessarily lead to effective teaching.


This chapter provides a useful overview of recent research on mathematics learning. A range of topics are discussed. These include a section on the learner, in which the learner is mainly looked at from the points of view of aptitudes and dispositions required to become a competent learner and problem solver in the domain of mathematics and the conception of the learner as a constructor of knowledge and competence. Another section discusses social and cultural contexts of mathematics
learning, including ethnomathematics, international comparison studies and the culture of the mathematics classroom. A section on instructional environments includes teachers' cognitions, designing powerful teaching-learning environments, integrating computers into instruction and reassessing assessment.


A review of research on teachers' knowledge. The authors make a distinction between teachers' knowledge of mathematics, teachers' knowledge of mathematical representations, teachers' knowledge of students and teachers' general knowledge of teaching and decision making. Studies suggest that teachers mathematical content knowledge is linked to both teacher behaviour in the classroom and to student outcomes. Teachers' knowledge of mathematical representations relates to how mathematics should be represented in instruction. If teachers do not have this understanding, it will be hard for them to teach pupils to understand mathematics. Knowledge of students and teaching are also considered important. The authors also discuss a number of models of teacher knowledge.


Lessons of two secondary maths student teachers were compared to those of the teachers they had been placed with, using field observations, pre- and post lesson interviews and planning materials. The novices' lessons were found to be less comprehensive than those of the experts, and their explanations were less conceptual. The experts more skilfully improvised activities and explanations around student questions and comments. These differences are explained by the assumption that
novices’ cognitive schemata for content and pedagogy are less elaborated, interconnected and accessible than those of experts.


The effect of teacher subject knowledge on pupil performance in mathematics was examined in 33 matched (by free school meal %, location and size) pairs of schools (9000 7th grade pupils) in which teachers differed as to their degree of subject knowledge as measured by the proxy variable of type of certification. In schools where the teachers had a higher certificate pupils performed better on the understanding and thinking skills scales of the Stanford Achievement Test, and slightly better on the knowledge/recognition scale.


Four teachers, classed as highly effective mathematics teachers, were studied. These teachers all viewed teaching as an activity involving constant growth and change, and had all significantly changed their teaching styles over time. They were characterised as reflective practitioners, who gave the impression of thinking carefully about what they were doing.


On the basis of four case studies of Californian teachers the authors argue that changing teachers practice must entail changes in teachers' beliefs and knowledge in three areas: teachers' knowledge and beliefs about learners, teaching and learning,
teachers' knowledge of mathematics and teachers' knowledge and beliefs about mathematics.


The self-concept of 16 male and female primary and secondary teachers was measured to study the possible influence of teacher's self-concept on how they perceive the nature of mathematics and their attitudes to teaching and learning mathematics (measured through individual teacher interviews). It was found that the low mathematical self-concept of some teachers was related to their negative experiences with mathematics as a pupil. High self-concept teachers were more motivated, more inventive and more creative about how to conduct maths lessons, while low self-concept teachers were more likely to be negative and to complain about lack of resources to implement what they considered to be effective ways of teaching mathematics.


A review of research on teacher beliefs and knowledge. The authors conclude that belief systems are dynamic and permeable mental structures, susceptible to change in light of experience. The relationship between beliefs and practice is also not a simple one-way relationship from belief to practice, but a dynamic two-way relationship in which beliefs are influenced by practical experience.

2d. Novice-Expert Studies, Constructivism and Other Miscellaneous Effective Teaching Research


In this chapter the authors look at some recent research in primary mathematics education. Both research focussing on analysing teaching/learning processes and research focussed on developing and testing instructional theories (such as the US Comprehensive School Mathematics Project and the Dutch Realistic Mathematics Education programme) are discussed.


This article reports on a 3-year ethnographic case study of 2 schools with alternative mathematical teaching approaches. One school used a back-to-basics textbook approach, the other used open-ended activities at all times. Using various forms of case study data, including observations, questionnaires, interviews and quantitative assessment, it was found that the two approaches encouraged different forms of knowledge. Students who followed the textbook approach developed procedural knowledge that was of limited use to them in unfamiliar situations. Students who learned mathematics in an open, project-based environment developed a conceptual understanding that provided them with advantages in a range of assessments and situations, and were able to use their knowledge in non-school settings.


This chapter focuses on the debate between propagators of the teaching of mathematics by applications and propagators of the dominance of pure mathematics (the 'new math'), from the point of view of a proponent of 'realistic mathematics education' supporting the use of applications. The chapter starts with historical reflection on the ongoing debate, and a review of the arguments for applications in mathematics education. Different aspects of using and applying mathematics are dealt with in detail. The case studies show different ways in which progress is made in implementing more applications-oriented curricula.


In this study two students were monitored to find out more about the metacognitive strategies they used in problem solving. It was found that students benefited from collaborative work in which they could adopt complementary metacognitive roles, but that their progress was sometimes impeded by unhelpful social interactions.

This volume contains contributions focussing on issues in mathematics education research rather than on classroom practice. Chapters generally provide an overview of theory or research to date and set out a research agenda for the future.

Topics discussed by contributors include teaching for higher order thinking, expert-novice studies, computer usage and cross-cultural studies.


A sample of 12 seventh grade students of average ability was studied to examine factors that impede or enhance mathematical problem solving. Factors found to impede problem solving were: lack of an experiential framework, imposition of unrequired restrictions (students imposed their own, non-required restrictions when solving a problem), lack of individual monitoring or regulating of cognitive activity and unproductive beliefs. Factors that enhanced problem solving were: group collaboration, group monitoring and social norms in small group problem solving.


6 second-grade classroom teachers were instructed in alternative ways of teaching place value, which emphasised constructing relationships between place value and computation rather than the traditional textbook-based approach used in control classrooms that emphasised practice. Classroom observation showed that in experimental classrooms pupils received fewer problems, spent more time with each problem, were asked more questions requesting them to explain and describe alternative strategies, talked more using longer responses and showed higher gains on a test administered before and after the experiment.
An overview of research on learning and teaching with understanding, grounded in a psychological framework that sees understanding as based on the assumption that knowledge is represented internally, and that these representations are structured. The authors consider a mathematical fact or idea to be understood if it is part of an internal network. Understanding is then produced as new information is connected to existing networks or as new relationships are constructed between previously disconnected information. The authors also discuss the consequences of understanding mathematics, such as the fact that it promotes remembering as well as reducing the amount that needs to be remembered. In the second part of the review the authors then turn to teaching methods that may promote understanding. These include the importance of making connections between mathematical ideas explicit for students, and correcting students' misconceptions.


This book describes a two-year research study to evaluate the implementation of the mathematics National Curriculum at Key Stages 1, 2 and 3. The development of the National Curriculum is discussed in order to provide the context for the study, followed by a description of the research focussing on teachers' use of commercial mathematics schemes, teachers’ perceptions of sequencing and progression in the mathematics National Curriculum, and different teacher 'readings' of the 'Using and Applying Mathematics' text, which was designed to introduce problem solving and investigation into the curriculum. In a final section policy implications of the study are discussed.

4 expert teachers and 2 novices were studied to examine factors contributing to expertise. Elements found to be of importance were: rich agendas (= lesson plans), consistent but flexible lesson structures, and explanations that meet the goals of clarifying concepts and procedures and having students learn and understand them.


A study of 306 3rd through 6th grade highly able pupils, who started off with more mathematical knowledge than their peers, found that these pupils made very strong progress when placed in a flexibly-paced program (individualised learning pace) as opposed to an 'age appropriate' curriculum.


Authors suggest ways of improving mathematics learning of learning disabled students. They suggest that educators should attempt to incorporate findings from various paradigms in an undogmatic fashion rather than reforming for the sake of reform, accommodate learner characteristics and needs, and implement practices based on solid research foundations.

In this chapter the authors review a number of factors contributing to mathematics teaching and learning. In a first section they discuss theories of mathematics learning, with particular emphasis on cognitive theory. Subjects covered include analysis of pupil errors and the organisation of memory and metacognition. Implications for teaching are given. In a second section mathematics teaching is discussed. Topics include assessment, allocated and engaged time, the developmental part of the whole-class interactive lesson, instruction in small groups, teachers' planning and decision making, teaching strategies and coping with individual pupil differences.


Two years after an intervention programme designed to promote pupils' formal operational thinking, the achievement of students initially 12 years of age was tested by their results in GCSE examinations. Although the intervention was set in the context of science, boys' achievement in mathematics (as well as the achievement of both genders in science) was significantly higher than that of control group students. The authors argue that this evidence supports the interpretation that students' increased achievement was caused by increased general intellectual capacity and not just by improved domain-specific skills.


The author presents a teaching model based on constructivist principles, using data from a classroom teaching experiment. The model (Mathematical Teaching Cycle) is based on teaching directed by the teacher's conceptual goals for his pupils, and
incorporates aspects such as teacher knowledge, thinking, decision making and activity.


A theoretical article that looks at a constructivist model of teaching in which the teacher's knowledge of pupil's thought processes is central. The teacher's role is essentially one of posing situations to allow pupils to develop independently but interactively their mathematical knowledge.


Discusses problems involved in supporting pupils' meaning-making of new mathematical concepts, which, according to the author, can become subsumed by ritualised communication patterns in typical mathematics lessons.


In this article the author reviews her experiences in working on two mathematics programmes teaching pupils the concept of variables with LOGO. These experiences are theoretically related to Vigotsky's learning theory.


Describes a problem-centered teaching approach in second and third year classes in one school district. Comparison of pupils who had received two years, one year and no problem-centered teaching found that students having received two years
of this type of instruction had significantly higher scores on standardised achievement tests, better conceptual understanding and more task-oriented beliefs for learning mathematics than pupils taught using textbook instruction. These differences remained after classes returned to textbook instruction following the experiment.

3. Mathematical Content and Curriculum


This edited book presents a number of perspectives on children's thinking in school mathematics, and the way this knowledge might improve teaching. In the first chapter there is an overview of the way the development of theories of children's learning have influenced the way mathematics is taught, and the role of changing technologies in this. In the second contribution early experiences children have in learning mathematics are considered, and ways in which this learning can be enhanced are discussed. Investigational activities with 10-11 year old children are the focus of the third contribution. Two chapters focus on the importance of language in learning mathematics, the first discussing the role of talk, the second the meaning of symbols. Calculators, information handling and algebra are the subjects of the remaining chapters.


Two groups of adolescents with learning difficulties in mathematics were compared on their ability to generate solutions to a contextualised problem after being taught problem-solving skills under two conditions, one involving standard word problems, the other involving a contextualised problem on videodisc. Both groups of
students improved their performance on solving word problems, but students in the contextualised problem group did significantly better on the contextualised post-test and were able to use their skills in two transfer tasks that followed instruction.


This review contains seven sections. First, students' performance in geometry is briefly summarised as a background to the research. Second, research on three major theoretical perspectives on the development of geometric thinking - Piaget, the van Hieles and cognitive science - is reviewed. Third, the establishment of truth in geometry is discussed, highlighting both theoretical and empirical work. Fourth, the relationship between spatial thinking and mathematics, the nature of spatial reasoning and imagery, and attempts to teach spatial abilities are considered. The fifth section reviews representations of geometric ideas, including issues related to concepts, diagrams, manipulatives and computers. Sixth, group and cross-cultural differences are examined. Finally, broad conclusions are drawn from the research corpus.


In this chapter the authors discuss what they consider to be substantive issues related to the design and development of new algebra curricula. They focus on potential ruptures between algebra and concrete mathematical activities that are often used as a basis for teaching algebra. The authors emphasise the importance of mathematical sign systems for communicating and learning algebra and discuss, from this perspective, the use of computer-based environments. The main thesis of the chapter centres on the
idea that algebra curricula have to take into account the teacher and teaching as well as curricular materials.


Review of research on whole number addition and subtraction. First the real world domain of whole number addition and subtraction situations is outlined. Then the developmental progression of conceptual structures that children between the ages of 2 and 8 construct to interpret and solve these situations is described. The unitary conceptual structures built for numbers are described first, and then the multi-unit conceptual structures built for multidigit whole numbers are discussed. A vision of what might be possible in pre-school and primary school classrooms is then summarised.


The author looks at the issue of how to teach students abstract mathematical knowledge. He claims that concrete modelling or the use of manipulatives is not helpful in this respect, as they do not help students attain mathematical insight. The author presents an alternative approach called realistic mathematics education, which is based on Freudenthal's theories and sees mathematics as mainly involving formalising, generalising and the a posteriori construction of connections. It is this generalising, as opposed to transfer, that is seen as the key to teaching children abstract knowledge. The chapter contains worked-out examples.

The strategies children use to solve simple arithmetic problems was studied in over 100 7 to 12 year old British children, using structured and open interviewing techniques. Two approaches were found to underlie the strategies used: counting (procedural strategies) or using selected known knowledge (deductive strategies). It appears that while low ability children only use counting, above average children call on both strategies. According to the author, for some children relying on procedural methods does not encourage in them a need to remember, while deductive strategies enhance the ability to remember basic facts that children can build on so they no longer need to remember new ones.


In the first part of this review, the range of applications of multiplication and division is set out, and the corresponding variety of external representations for the operations is illustrated. In the second part, the psychological complexity behind multiplication and division is demonstrated further by reviewing a number of theoretical frameworks that treat the subject from several different perspectives. There follows a broader framework, the key points of which are that:

a. multiplication and division model many distinguishable classes of situations;
b. a fundamental conceptual restructuring is necessary when multiplication and division are extended beyond the domain of positive integers.

The future role of computers is also discussed.


This book section aims to provide a theoretical and historical setting for proof and proving in mathematics, in order that this context may inform classroom teaching of
proof. The author also offers insights into using proof for furthering mathematical understanding, for which some types of proof are more suited than others.


Summary of a research project aimed at examining the impact of the National Curriculum on high attaining year 10 students' views of and competencies in mathematical proof. Two questionnaires were designed, a proof questionnaire containing questions on pupils' views on proofs and a number of proof items, and a school questionnaire designed to collect information of the pupil's school and teacher. 2459 high attaining year 10 students in 94 classes in 90 schools took part in the survey. Main conclusions were that:

1. High attaining year 10 students performed poorly in constructing proofs;
2. Students' performance in algebra was considerably better than in geometry;
3. Most students appreciate the generality of a valid proof;
4. Students are better at choosing a valid mathematical argument than at constructing one, although their choices are often influenced by non-mathematical factors;
5. General mathematical ability has a consistent influence on students' views of proof and their competencies in proving;
6. Students' views of proof and its purposes account for differences in their responses;
7. In algebra, there are significant gender differences, with girls constructing better proofs than boys and choosing different forms of argument;
8. Teacher characteristics are not associated with students' competencies in proving;
9. A range of school and curriculum factors are associated with performance;
10. After taking into account all the factors found to influence student performance, there remains unexplained variation in the responses of students attending particular schools.

Meta-analysis of research on problem solving. Successful problem solving was associated with high mental ability, especially in forming analogies; with attitudes towards mathematics and mathematical self-concept; high socio-economic status and being male from grade 9 onwards. Better performance on problem-solving tasks was also linked to the problem being set in a familiar context or including a picture, and lower performance with the inclusion of extraneous data (data not relevant to the problem). Other factors (such as concrete contexts and readability) were not significant. Receiving instruction in problem solving skills was positively related to performance, as was training in specific problem solving subskills.


Three perspectives about the possible roles of shape and space are discussed: 1. Interacting with real shapes in space, 2. shape and space as fundamental ingredients for constructing a theory and 3. shapes or visual representations as a means for better understanding of concepts, processes and phenomena in different areas of mathematics and science. Students' ability to visualise real objects, mathematical concepts, processes and phenomena is nowadays considered a mathematical activity like computing or symbolising. Nevertheless, according to the authors, unlike numerical or algebraic education, visual education is often a neglected area in curricula. The authors therefore describe three projects that invested efforts in a systematic development of visual education.

This review begins with an historical analysis of the development of algebra, followed by a description of the content of school algebra and a discussion of the psychological demands made on the algebra learner by the mathematical content. Then a brief overview of the teaching perspective is provided. Major findings from research on the learning and teaching of algebra are discussed in relation to the historical-psychological framework of the prior sections. The chapter concludes with a synthesis and suggestions for future research.


In this study two programmes for teaching mental addition and subtraction in the Dutch 2nd grade (N=275) are compared. The goal of both programmes is greater flexibility in mental arithmetic through the use of the empty number line as a new mental method. The programmes differ in instructional design to enable comparison of 2 contrasting instructional concepts. The Realistic Programme Design (RPD) stimulates flexible use of procedures from the beginning by using realistic context problems. The Gradual Programme Design (GPD) has as its purpose a gradual increase in knowledge through initial emphasis on procedural computation followed by flexible problem solving. It was found that whereas RPD pupils showed a more varied use of solution procedures than GPD pupils did, this variation did not influence the procedural competence of the pupils. The empty number line appeared to be a very powerful model for the learning of addition and subtraction.


This paper discusses important aspects of teaching number at key stages 1-3. The report discusses four main items: the understanding of place value in whole numbers and decimals, developing methods of computation (with an emphasis on
mental strategies), solving numerical problems and using and applying mathematics in relation to number. Pupils' difficulties are discussed and methods to help overcome these presented in all three main sections.


This book section looks at the role of data handling in mathematics curricula. The historical roots of contemporary data handling emphasis are discussed first, followed by a review of a number of national reform efforts that have catalysed an interest in data handling. Various data handling curricula are discussed. Special attention is given to the use of technology in data handling, the importance of the professional development of teachers of data handling, and to some issues for research in the teaching and learning of data handling.


Various aspects of estimation and number sense are discussed in this book chapter. How people estimate computations and what abilities are related to estimation ability, how computational estimation concepts develop and the effects of instruction on computational estimates are reviewed. Mental computation and number sense, both related to estimation, are discussed next. A section on measurement estimation follows, with ability to estimate and instruction on estimation two major topics. In a final section assessment issues relating to estimation are discussed.


Looks at the way fractions evolve from the everyday experience of fair sharing. The author contends that it is better to start the learning of fractions from everyday
situations which are then 'mathematised', and that quantities and magnitudes play a mediating role in the move from everyday experience to the concept of fractions as operators. The chapter presents an approach to teaching that takes this role into account while aiming to take them one step further to the conception of fractions as operators. This method belongs to the 'realistic mathematics education' approach advocated by the Dutch Freudenthal Institute.


This useful edited book gives an overview of the teaching and learning of number to young children. The book has four parts: in part one the numerical understanding and beliefs of pre-school children are discussed, in part two the important role of counting in number development is explored, while in part three the role of written number work (e.g. when to start doing it and the relationship between written and mental algorithms) is discussed. In the final section a number of perspectives on teaching number are given, including the role of language, the use of calculators and how to develop young children's counting skills.


This book section begins with a summary of the general characteristics and principles underlying the ongoing world-wide reform of mathematics education. It then goes on to document and illustrate how these general principles permeate a major domain of the mathematics curricula for the elementary school: number and arithmetic. Five related topics within this domain are discussed, namely: number concepts and number sense, the meaning of arithmetic operations, mastery of basic arithmetic facts, mental and written computation, and word problems as applications of numerical and arithmetical knowledge and skills. The final section lists some remaining issues and
tasks for further curriculum research and development in the domain of number and arithmetic.
4. Child Development and Mathematics


This edited book presents a number of perspectives on children's thinking in school mathematics, and the ways in which this knowledge might improve teaching. In the first chapter there is an overview of the ways the development of theories of children's learning have influenced the way mathematics is taught, and the role of changing technologies in this. In the second contribution, the early experiences children have in learning mathematics are considered, and ways that can enhance this learning are discussed. Investigational activities with 10-11 year old children are the focus of the third contribution. Two chapters focus on the importance of language in learning mathematics, the first discussing the role of talk, the second the meaning of symbols. Calculators, information handling and algebra are the subjects of the remaining chapters.


Study of a reception class of 16 children. It was found that pupils already possessed a variety of mathematical knowledge at entry into reception, the high attaining children already being well towards mastery of level 1 of the National Curriculum in this group. The low attaining children brought to school a range of informal competencies and a less stable conventional knowledge. The author points out some teaching implications of these findings.


The development of children's understanding of mathematical relations and their grasp of the number system is described. It is argued that children easily recognise one-way part-part relations but that the number system at first causes them difficulty.
Children’s relational understanding allows them to solve addition and subtraction problems fairly well when these deal with simple increases and decreases in quantity. It also helps them to make proportional judgements when these involve part-part relations. However, problems that involve relations between parts and wholes are at first extremely difficult. The review also deals with the effects of context and shows the considerable aptitudes that are handed on to children in informal settings.


The author looks at whether or not children have mathematical understanding before they go to school. The author concludes that young children are quite good at learning to count, but have little grasp of the meaning of number words. They do have a good grasp of what is involved in sharing, in particular of the importance of one-to-one correspondence in sharing. This understanding allows them to take a first step towards learning about division. They can also add and subtract small quantities.


The authors contend that an understanding of students’ thinking can provide coherence for teachers’ pedagogical content knowledge and their knowledge of subject matter, curriculum and pedagogy. The authors propose a model of children's thinking that could inform teacher's teaching.

The strategies children use to solve simple arithmetic problems was studied in over 100 7 to 12 year old British children, using structured and open interviewing techniques. Two approaches were found to underlie the strategies used: counting (procedural strategies) or using selected known knowledge (deductive strategies). It appears that while low ability children only use counting, above average children call on both strategies. According to the author, for some children relying on procedural methods does not encourage in them a need to remember, while deductive strategies enhance the ability to remember basic facts that children can build on so they no longer need to remember new ones.


This book presents a theory of the growth of children's mathematical knowledge. The authors aim to show how children think about mathematical problems and what mathematics means to them. In studying the way children's mathematical reasoning develops, the authors stress three concepts that lie at the heart of mathematical learning: children have to pick up a great deal of knowledge about logical relations, they have to learn conventional mathematical systems, and they have to learn that certain mathematical relations which they originally regarded as tied to very specific situations have much wider uses. The authors also emphasise the prior knowledge children take to school with them.


This book attempts to link findings from cognitive science to mathematics instruction. The first three chapters discuss some central themes in cognitive science. Chapter 1 provides a broad description of cognitive science, and reflects upon what the
major contributions of the science to mathematics education could be. The next two chapters provide more detail. Chapter 2 describes basic work on memory and information processing. An overview of problem solving theory and research is presented, the educational implications of which are then discussed. In Chapter 3 a description of cognitive representations is presented along with their implications for mathematics instruction.

Chapters 4 through 6 establish contexts for research and development in mathematics education. In chapter 4 the role and functions of computer-based technology in mathematics education are examined. In chapter 5 the literature on problem formulation is reviewed, and in chapter 6 classroom reality is described from the teacher's point of view.

Chapters 7 through 9 summarise particular bodies of work in cognitive science with direct instructional implications. In chapter 7 the literature on 'bugs' (students' systematic errors in algorithmic procedures) is summarised. The subject of chapter 8 is metacognition, and in chapter 9 research on students' difficulties with algebra, leading to the development of computer-based remedial tutorial systems, is outlined. In the final two chapter a mathematician and a mathematics educator reflect on the preceding chapters.


This article discusses young children's development and knowledge of ratio, proportion and functional reasoning. The authors conclude that intuitive bases for this knowledge are available to small children. Two different types of intuition seem to exist: direct apprehension (speed, density etc. are considered attributes of objects in their own right) and protoquantitative reasoning (covariation between different things). It
would seem that even quite young children are capable of this, as long as they are not asked to quantify it.


Evidence concerning intuitive foundations for fraction learning was obtained in a study of early development in proportional reasoning. 20 children aged 5 to 7 were given problems constructed so as to differentiate between reasoning based on relations between one part and another. The participants were able to use part-whole relations to compare proportions by 7 years of age. In addition, a developmental shift towards increased reliance on part-whole reasoning was observed in children's responses to conflict problems that pitted part-whole and part-part matches against each other.


Reviews research on elementary addition and subtraction word problems carried out at Leuven University. It was found that some erroneous thinking processes in young children were due to deficiencies in children's conceptual knowledge base, while others resulted from the application of superficial coping strategies (e.g. always adding a given number without trying to understand the problem). Further, it was found that the effect of the semantic structure of word problems was strong, for both addition and subtraction problems, and finally it was found that the semantic structure of word problems interacts with other task characteristics in complex ways.

A review of research on solving one-step arithmetic word problems. A classification of word problems is given and pupils' informal and formal problem solving strategies are discussed, as well as the developmental stages that children have to go through before being able to solve word problems. The authors also point to lessons from this research for the construction of new problem-solving materials.
5. **ICT and Calculators in Mathematics Teaching**


This book section reviews the effects of computer-based learning environments (such as LOGO) and attempts to set out where such environments are heading. The authors look at artificial intelligence (and its limits) in mathematics, and discuss the problems encountered with computer use in the classroom. They stress the need for teacher development to enable effective use of computer-based learning environments by teachers.


Report on a number of studies done to examine the impact of the use of Integrated Learning Systems in UK schools. Main findings were that use of ILS had a marked and favourable effect on pupils' attitudes, motivation and behaviour. It was not clear, however, whether these positive attitudes generalise beyond pupils' experience with ILS to their general attitudes towards school. The effects of ILS on pupil achievement and learning were less clear, however, and relying exclusively on ILS for preparation for Key Stage 3 tests or GCSE exams was found to have negative effects. Teacher preparation (as well as ILS) seems essential here.


Looks at group learning with the LOGO computer programme. Eight groups of 12 students (aged 9 to 12) were studied while doing two group tasks associated with LOGO. Comparing pre- and post-test results, pupils who had taken part in the activities had higher gain scores than control group pupils (although experimental pupil's test scores were higher to start with). Different types of task organisation were found to be
differentially effective with different tasks, an integrated style working better for one of the two tasks, a fragmented style working better for the other task, which suggests the importance of adapting task organisation to the goal to be attained.


Meta-analysis of 79 research reports on the use of calculators in mathematics lessons. In all grades except for grade 4 calculator use (in concert with teacher instruction) had a weak positive effect on pupils performance in basic pencil and paper skills. Calculator use also had a positive effect on pupil attitudes and self-concept.


This chapter has three aims: first, to present an analysis of some of the writing and research on ICT and primary mathematics; second, to identify what the authors see as some trends in the way computer programs for primary maths have developed, and third, to outline the implications of these for classroom teachers. Two main strands of computer programs are identified: drill-and-practice programs such as *Successmaker*, and constructivist or understanding-led approaches to software design such as LOGO. Neither have so far been able to demonstrate convincing positive effects on pupil learning. This may change as software development moves forwards, however. The authors conclude with a number of recommendations for teachers, based on the premise that the choice to use a particular software program must always be based on whether using ICT in a given situation will be of benefit to attaining the teacher's
specific teaching goals.


Review of research on technology and mathematics education. A perspective on the new technologies is offered that attempts to unify the unique phenomena that these technologies offer. This perspective is applied to make sense of the different forms that computing environments take, and especially how they change the processes of learning and using mathematical ideas. A review and a critique of the various uses of artificial intelligence in mathematics education that draws contrasts with the field of medicine is presented. The last major section considers issues of implementation, not to provide concrete tactics for dealing with particular situations, but rather to provide a general strategy.


28 second grade pupils learned simple multiplication by either generating (computing) the answer themselves or by reading the answer from a calculator display. Post, pre and retention tests showed that generating the answer was highly effective for children with a low prior knowledge of mathematics, and ineffective for children with a high prior mathematical knowledge. On the retention test children taught by generating
the answer themselves showed no loss over time, whereas children taught by reading the calculator answer did show a loss in efficiency over time.


This study of 286 pupils in eight fifth grade elementary school classrooms was designed to assess the combined and separate effects of instruction via a computer and video programme on students' learning processes, intrinsic motivation and achievement in geometry. Both a programme using video and a programme using the computer achieved better results. Using the two combined, however, yielded learning outcomes lower than both 'one-medium' conditions and similar to the 'no-media' condition. Students exposed to the combined programme did gain more intrinsic motivation than students in the no-media condition.


Describes a programme developed to allow gifted students to complete advanced courses in mathematics several years before they would usually do so, using computer software. 15 students participated in the programme, and achieved high scores on advanced placement exams.


In this chapter the idea of personal computational technology in mathematics education is explored, and different portable devices, from the arithmetic calculator to the laptop computer, are described and appraised in terms of their potential as an
individual resource for classroom use. The relatively modest impact of calculators on mathematics education is documented, and two examples of ways in which calculators have been integrated into classroom mathematics are sketched. Findings from research into the effects of calculators are summarised and evaluated. In the final section various ways of enshrining calculator use in processes of mathematical thinking and teaching mathematics are identified and discussed.


This study examined the use of mental, written and calculator strategies of numerical calculation by 56 year 6 primary school pupils. Pupils were drawn from neighbouring schools with very different traditions to the teaching of number. The 'calculator-aware' approach employed in one group of schools encouraged pupils, from an early age, to develop informal methods of calculation and to use calculators to explore number and execute demanding computations. Such pupils were found to make greater use of mental computation, particularly of multiplication strategies based on distribution and compensation. These aspects of performance were found to be more strongly associated with curricular experience than with number-concept attainment.


Data from 6 primary schools (year 6) was used to investigate the use of the calculator as a cognitive tool. Three of the schools had taken part in the Calculator-Aware Numbers project (CAN). It was found that while the calculator can act as an alternative to often poorly understood written column methods, the viability of this depends on the availability of appropriate mental schemes.
Longitudinal study of pupils from reception to year 6 in 6 schools, three taking part in the Calculator-Aware numbers project, three others matched as control schools. Results (using Key stage SAT’s and further tests at year 11) indicate no long-term effect of participation in the programme on test results.


This paper reviews research on calculator use at key stages 1-3. An overview is given of calculator use in England before and after the introduction of the National Curriculum. International comparisons are reviewed, suggesting no strong relationship between calculator use and achievement. A review of research of calculator use in the primary sector concludes that there are no strong effects of calculator use in either a positive or a negative direction. A number of intervention studies (such as CAN) in which calculator use was encouraged have proved successful, but other factors in these programs are likely to have been more directly responsible for these results. The report also suggests a number of ways in which calculators can be used effectively in the classroom.
6. International Studies of Mathematics Achievement


This book chapter provides an international examination of current approaches to the preparation of mathematics teachers. The authors give an overview of mathematics teacher education in four countries - England, France, Germany and the United States, focussing both on the structure and contexts of teacher preparation in each country and on characteristic features of each country's system. The authors then single out France and the United States for further comparison, looking in particular at teacher education students, teacher educators themselves and the curriculum of teacher education.


Cultural differences in the daily lives of 578 11th grade students in the US, Taiwan and Japan were examined. Chinese students spent significantly more time than American students engaged in academic pursuits, such as after-school classes and studying. Japanese students did not spend more time than American students on after school academic activities, but did spend more time at school. American students spent more time working and socialising. Differences in adolescents' time use were related to both cross-cultural and individual differences in mathematics achievement.


Reports on the TIMSS project which shows relatively poor achievement by English nine and thirteen year olds on overall mathematics achievement tests, but with strength in certain areas such as geometry. Various educational factors are investigated to account for these country differences, such as time spent on mathematics, homework utilisation, use of calculators and classroom organisation.

125 4th grade Canadian pupils and 128 Hong Kong Chinese pupils were compared on mathematics achievement (maths test scores) and perceptions of maths competence as rated by parents, teachers and pupils. Hong Kong Chinese pupils outperformed Canadian pupils on the test. However, ratings of children's scholastic/mathematical competence were higher among Canadian children. There were also a number of differences in children's self-concepts, Canadian children rating physical appearance and social acceptance as the most important factors, while for Hong Kong children the most important facets were behavioural conduct and scholastic competence.


Classroom observation was used to study differences between lower secondary school mathematics classrooms in the three settings. It was found that while all lessons in Beijing and Hong Kong followed a whole-class pattern (teacher reviews previous lesson, develops topic, pupils practise, students present their work and lesson is summarised), in London almost half the lessons used an individual instruction approach (students work on their own while teacher goes round helping them). In Beijing mathematics lessons were more structured and there was greater stress on mathematical content, while in Hong Kong the emphasis was on practising mathematical skills. In London the stress was on students doing mathematics on their own. Students in Beijing were asked more questions and spent less time off task.


This paper describes reforms in mathematics teaching in the London borough of Barking and Dagenham influenced by continental whole-class teaching methods.
Teachers from the borough visited Swiss schools (Switzerland being a high performer in comparative studies of mathematics achievement) and reforms based on the structured, whole-class interactive approach emphasising mental arithmetic were implemented in the borough.


This study was conducted to explore performance on variety of mental computation tasks using two presentation formats (oral and visual). 2000 students at each of four grade levels (from grade 2 to grade 9) in 6 classes each country were selected. Results indicate wide within-country performance differences, but significant between-country differences were also found. Japanese students perform at a higher level, particularly in the lower grades, the difference narrowing in the higher grades and the difference with the Australian pupils becoming non-significant by grade 8.


A comparison of the treatment of addition and subtraction of signed whole numbers in seventh grade textbooks in the US and Japan. It was found that the Japanese textbooks contained more worked-out examples and relevant illustrations, whereas the American textbooks contained more irrelevant attention-grabbing illustrations. The Japanese textbooks also devoted more space to procedures for working out examples, to meaningful instructional models emphasising multiple representations of how to solve worked-out examples, and to inductive organisation of
material beginning with familiar situations and ending with formal statements. US textbooks devoted more space to unsolved exercises.


This cross-national study found evidence that cognitive representation of number may differ depending on the language spoken, and may partially account for performance differences between Asian and non-Asian groups.


In this cross-national study of first grade classrooms in the US, Japan and Taiwan, in which 311 lessons were observed, it was found that Asian teachers asked significantly more questions about conceptual knowledge and problem solving strategies than did US teachers. Chinese teachers asked significantly more questions embedded in a concrete context than did US teachers. These findings may help explain performance differences between these countries.


This paper looks at the issue of the attainment of summer-born children. The English age-bound practice is compared to continental methods of deferring entry to primary school for slow developing children, so as to make the class more homogeneous, thus facilitating whole-class teaching. In a sample of around 600 pupils aged 9-10 in England and Switzerland, it was found that deferred pupils in Switzerland
performed close to the average of the class in which they had been placed, and that the variability of pupils' attainments in maths had been much reduced, to only about half that of the English classrooms. Thus greater flexibility in age of entry might aid whole-class teaching, it is argued.


Reviews the evidence on international comparisons focusing particularly on mathematics achievement of countries from the mid 1960’s to 1996. It is concluded that England exhibits poor levels of attainment, a wide range of attainment and considerable variation in the opportunity-to-learn mathematics. Speculations from ongoing comparative work are given to account for these findings.


Review of the First and Second International Mathematics studies, as well as a number of smaller scale international studies.


This study explored the effects of teachers' evaluative feedback on students' perceptions of ability. 758 Chinese students from elementary schools, high schools and a university rated ability and effort of two hypothetical students who achieved identical results on a maths test but received different evaluative teacher feedback. They also rated the teacher's perception of their own effort and ability in a similar situation after a real maths test. Contrary to findings among Western students, ability and effort were
positively correlated. The importance of effort among the Chinese students appeared to mitigate the effect of teachers' feedback on students’ ability.


Using data from SIMS, the authors look at reasons for the lower performance of American as opposed to Japanese pupils in mathematics. It was found that Japanese teachers are better at restricting the ability range in their classrooms. Japanese teachers tend to teach the whole class more, while American teachers let students work on their own more.


Data from the Second International Mathematics Study are re-examined in this edited book. The focus is on the US data, both in comparison with other countries and within the country. Various contributors touch on a variety of issues using SIMS data, such as opportunity to learn, textbook coverage, teaching problem solving, a comparison of American and Japanese teaching, gender differences across countries, and methodological issues in mathematics education research.


The author points to a number of differences between Japanese and English Classrooms that may account for the difference in performance of pupils from these two countries on international tests. Japanese teachers tend to teach the whole class, while English teachers encourage children to progress at their own pace, thus contributing to
an increase in the range of attainment. Japanese teachers by contrast emphasise keeping the whole class together. Japanese teachers have a high level of professional competence, and their teaching is thorough and rigorous. In Japan the performance of the group is seen as more important than the performance of individuals, who do not compete against one another.


The author discussed a number of differences between English and Japanese classrooms that may contribute to the better performance of Japanese pupils in international tests at the secondary level. In Japan, there is a high degree of state control over education in which all pupils are expected to reach certain common educational goals. This in contrast to the higher degree of freedom in England, where pupils are allowed to develop at their own pace to a larger extent. This, along with a lack of facilities to help stragglers, is likely to widen the attainment range in England. Cultural differences in the value placed on education and mathematics by society may also play an important part in the attainment differences.
7. Teacher Training and Professional Development in Mathematics


The authors look at the relationship between the developing field of mathematics education research and teachers' professional knowledge of the field. The authors look at different orientations in teacher education, and their implications for teachers' views of mathematics education. The authors look at ways of introducing teachers to the research base.


In this experiment knowledge from teacher effectiveness was used for staff training in urban elementary schools. Training took place during 5 days, 3 weeks prior to the beginning of term. Based on both trainer and staff self-reports and on observation of teachers a significant treatment effect was found on 4 out of 10 categories of effective teaching behaviour: academic planning and preparation, academic presentation, organising for classroom management and presentation of rules and procedures.

This article reviews the relationship between teacher education and mathematics teaching, and attempts to provide some answers to the question of how to train teachers to teach for more understanding in mathematics classrooms.


This article describes characteristics of a preservice teacher training programme designed to prepare teachers for the 'realistic approach' to maths in the Netherlands.
8. The Effects of Parental Background, Gender and Ethnicity


In a sample of 957 grades 3 to 7 elementary school children boys reported higher scores on descriptive and evaluative statements on mathematics. Mathematical self-concept declined over time for both genders.


58 first grade children solved addition and subtraction problems individually and in groups of three in October, January and May. The children's strategy use was assessed, as well as their metacognitive knowledge for mathematics strategies and their rationale for the use of different mathematics strategies. Gender differences were found: girls were more likely to count on their fingers or use counters (overt strategies), boys were more likely to use retrieval (from memory) to solve addition and subtraction problems. All children were more likely to use covert strategies and retrieval and less likely to use overt strategies when they were working in groups. Metacognition was a significant predictor of strategy use.


Using data from the US National Longitudinal Survey of Youth (over 7000 children in the '86 and '88 waves), the relationship between mathematics achievement and a number of background factors was tested. It was found that home environment,
parental socio-economic status and maternal cognitive test scores all had a significant effect on children's test scores at ages 5 and 9. It was found that about 25% of the relationship was due to maternal cognitive ability, while 75% was due to home environmental factors.


Using three samples of Northern Irlelish pupils taken in the 80's and 90's, the authors look at the effects of gender and co-educational vs. single sex schools, taking into account pupil and family background factors such as religion, S.E.S., verbal reasoning band etc. Gender and school effects are small, and girls seem to be closing the performance gap on boys in these samples. Family S.E.S. and ability are the main factors affecting attainment and examination entry.


This study examined the effects of a home-based parent involvement (PI) intervention and a reciprocal peer tutoring (RPT) intervention on the self-concept and mathematics achievement of 72 4th and 5th grade students experiencing problems with mathematics. Students were randomly assigned to three conditions (PI, RPT or PI+RPT) and practice control. Students self-concept reports showed that students in the PI and PI+RPT groups had higher self-concepts than control group pupils. Students in the PI+RPT group perceived themselves as more socially confident than students in the control group. PI+RPT students also had significantly higher scores on a standardised maths test than control group students, and also displayed a higher level
of accurate computations than PI or control group students.


Review of research on the relationship between gender and mathematics achievement published in the Journal for Research in Mathematics Education. Main conclusions are that gender differences may be decreasing; but that they differences still exist(ed) in the learning of complex mathematics, personal beliefs in mathematics and career choices that involve mathematics. Gender differences in mathematics vary by school, socio-economic status, ethnicity and teacher. Teachers tend to structure their classrooms in ways that favour male learning. Specific interventions can achieve equity in mathematics.


A meta-analysis of studies appearing between 1974 and 1986 found that gender differences in mathematics were small, and were decreasing over the years. The difference in favour of boys is higher in the upper years of secondary school and among high achievers.


Using a longitudinal data set of 1477 students in 48 classes (primarily grades 4 - 6) in Northern California, it was found that within-class assignment to ability groupings in school was influenced by pupil gender over and above pupils' performance in maths, with boys tending to be assigned to higher ability groups. Girls with high mathematical aptitude were less likely to be assigned to a high set than boys with similar aptitudes. Within-class grouping was found to have no effect on pupil performance compared to
A comprehensive review of research on mathematics and gender. A number of gender differences were found. Thus, in the US, males participate more than females in non-compulsory mathematics courses. Performance differences also seemed to exist, in that from the beginning of secondary school males frequently outperform females on standardised mathematics tests. Differences are especially marked on high cognitive level questions, and for high achieving students. However, these between gender differences are dwarfed by much larger within gender differences. The author also offers a number of possible explanations for the existing differences, including biological variables, environmental variables (school, teacher, peer group, parents and society), learner-related variables and cognitive variables (such as intelligence, spatial abilities, confidence, fear of success, attributions and persistence). Most important of these would seem to be subtle but consistent differences in cultural and societal pressures and expectations for females and males, which are frequently internalised by individuals and may lead to differing beliefs and expectations in areas critical to mathematics learning.


A sample (N=1170) of pupil's taking the Midland Examination Group's GCSE examinations in June 1994 was studied to determine the relationship between class size, gender, LEA/GM status and GCSE mathematics examination scores. It was not found that smaller class sizes were associated with better examination results, there being, on the contrary, a positive correlation between class size and results. This,
however, does not mean that larger class size leads to better performance, as the results could be explained by such factors as schools placing their weakest pupils in smaller classes. In the basic tier girls’ performance was significantly (but not strongly) higher than that of boys.


When a sample of 2586 academically talented students in grades 2-6 was given a test of mathematical reasoning ability, boys performed better overall than girls. The gender differences for mathematical reasoning ability appeared as early as 2nd grade in samples tested over a 7-year period, but varied somewhat according to mathematical subskills. There were no substantial gender-related differences on tasks requiring students to identify whether enough information was provided to solve a task. However, boys performed better than girls on tasks requiring application of algebraic rules or algorithms, as well as on tasks in which the understanding of mathematical concepts and number relations was required.


Classroom observation study of 36 fourth grade teachers in 15 schools, looking at gender differences in engagement during different activities. Pupils took a pre and post observation achievement test with low and high level items. Boys and girls did not differ in overall achievement on either the pre- or the post-test. On-task behaviour was significantly related to achievement among both genders. The amount of overall on-task behaviour did not differ across gender. Looking at different activities it was found that engagement in competitive activities was significantly negatively related to low level achievement of girls and slightly positively to that of boys. Engagement in cooperative
activities was significantly positively related to low level achievement among girls, and negatively related to low level achievement among boys. There was also a weak positive relationship between engaging in cooperative activities and high-level achievement among girls.


A three-wave longitudinal model tested factors affecting mathematics achievement and attitude among 3116 young adolescents. Home environment, motivation, exposure to mass media, instructional time and (weakly) peer environment (in order of significance) influenced mathematics achievement, while motivation, instructional quality and home environment influenced maths attitude (in order of significance). Both achievement and to a lesser extent attitude showed stability over time.


In this longitudinal study of over 2500 high school students a structural model of mathematics achievement was tested. A complex model in which home environment, taking an advanced course, classroom context, instructional time, motivation and peer environment (in order of significance) influenced achievement was found to fit the data. Mathematics attitudes were influenced by motivation, mathematics achievement, home background, instructional time, classroom context and peer environment (in order of significance). Both achievement and attitude showed stability over time.

This review discussed the relationship between mathematics achievement and background factors such as ethnicity, race, language spoken and social class. Among US students, Whites do better on standardised achievement tests than Hispanics, who do slightly better than African Americans. The gap in attainment between African Americans and Whites seems to have narrowed over time on basic skills, but not on higher level skills. Both achievement and attainment in mathematics are lower among pupils from low socio-economic status backgrounds, and a large part of the ethnic differences mentioned above can be accounted for by differences in socio-economic status. Language proficiency also seems to be somewhat related to mathematics achievement in that language. The author also discusses programs in the US that have attempted to alleviate these differences, such as Head Start, Sesame Street and Chapter 1 programs, many of whom seem to have positive short term effects, which are not always sustained. Classroom and school factors such as differential teacher expectations for different social groups and tracking, which might disadvantage low SES and ethnic minority groups, are also discussed. Direct instruction has been found to be an effective way of teaching basic skills to disadvantaged groups, as has cognitively guided instruction.
9. **Pupil Attitudes and Self-Concept in Mathematics**


In a sample of 957 grade 3 to 7 American elementary school children boys reported higher scores on descriptive and evaluative statements on mathematics. Mathematical self-concept declined over time for both genders.


This study examined whether male (n=25) and female (n=22) students of high mathematical ability use different solution strategies on maths problems that had previously yielded gender differences in correct responding using a structured interview format. Eight types of solution strategies were dichotomised as either conventional or unconventional in approach. It was found that female students were more likely than male students to use a conventional approach. Use of these conventional strategies was correlated with negative attitudes towards maths.


In a study of pupil perceptions towards ‘core’ subjects carried out amongst 48 Key Stage 3 pupils, mathematics was found to be more of a 'love-hate' subject than others, scoring high on both liked and disliked subjects scales.


36 first graders from 2 different school systems participated in interviews to determine their views of what it means to engage in mathematics and the rationale and
conditions under which they held such perceptions. The children were in classrooms that reflected the spirit of the current (constructivist inspired) reform movement in the US. Generally, the children perceived mathematics to be a problem-solving endeavour in which many different strategies are considered viable, and communicating mathematical thinking is an integral part of the task.


A meta analysis of the relationship between attitudes towards and achievement in mathematics based on 113 studies in primary schools. The overall effect of attitudes towards mathematics on achievement in mathematics was a barely statistically (and not substantively) significant .08, while the inverse relationship was non-significant. The relationship got stronger over time, however, being more significant in the secondary grades. The relationship was stronger among Asian and black students than among whites, but did not differ across gender.


This review of research first discusses alternative theoretical frameworks for research on affect. One of these, Mandler’s cognitive psychology-based approach, is singled out for further discussion, because it illustrates how research on affect can be incorporated into cognitive studies of mathematics teaching and learning. The chapter then presents a framework for research on affect. In the next section research on a number of topics from the framework is summarised. Student beliefs about mathematics have been found to affect their ability in various domains, while their self-beliefs are related to their performance in mathematics. Males tend to have more positive mathematical self-beliefs than females, and for both genders mathematical self-belief decreases through the primary school years. Attitudes are likewise related to
performance, though research suggests neither attitudes nor achievement are
dependent on one another, but rather they interact with each other in complex and
unpredictable ways. The authors also review research on emotions and other affective
constructs.

Muijs, R. D. (1997). “Predictors of Academic Achievement and Academic Self-
Concept: A Longitudinal Perspective.” *British Journal of Educational Psychology*
67: 263-277.

A two-year study using data from a sample of 1001 Flemish primary school
children (years 4 and 5) found that academic self-concept and academic achievement
were strong predictors of one another, controlling for background variables and stability
over time. Though both predicted each other, academic achievement was found to be a
stronger predictor of academic self-concept than academic self-concept was of
academic achievement.

Engagement in Classroom Activities and Sex-Related Differences in Learning

Classroom observation study of 36 fourth grade teachers in 15 schools, looking
at gender differences in engagement during different activities. Pupils took a pre and
post observation achievement test with low and high level items. Boys and girls did not
differ in overall achievement on either the pre- or the post-test. On task behaviour was
significantly related to achievement among both genders. The amount of overall on task
behaviour did not differ across gender. Looking at different activities it was found that
engagement in competitive activities was significantly negatively related to low level
achievement of girls and slightly positively to that of boys. Engagement in cooperative
activities was significantly positively related to a low level of achievement among girls,
and negatively to low level achievement among boys. There was also a weak positive
relationship with high-level achievement among girls.

This study examined the relationship between academic task values (for mathematics and language) and perceptions of personal social satisfaction for children in classrooms using a cooperative, interactive learning structure or in regular classrooms. 162 children in six form 2 class (aged 13-15) participated. Three classes were taught using cooperative learning strategies. Using questionnaire scale results, it was found that task values for engagement in mathematics and language were higher, and perceived costs lower in classrooms using a cooperative goal structure. This was also associated with higher social satisfaction in students.
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